

7-4 Remainder and Factor Theorems

Solve.

$$x^3 + 4x^2 - 15x - 18 = 0$$

If $x - 3$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -15 & -18 \\ & & 3 & 21 & 18 \\ \hline & 1 & 7 & 6 & 0 \end{array}$$

$$x^2 + 7x + 6 = 0$$

$$(x+6)(x+1) = 0$$

$$\{+3, -6, -1\}$$

Solve.

$$x^3 + 7x^2 + 2x - 40 = 0$$

If $x - 2$ is a factor.

$$\{-5, -4, 2\}$$

$$\begin{array}{r|rrrr} 2 & 1 & 7 & 2 & -40 \\ & & 2 & 18 & 40 \\ \hline & 1 & 9 & 20 & 0 \end{array}$$

$$x^2 + 9x + 20 = 0$$

$$(x+5)(x+4) = 0$$

Solve.

$$x^3 - 2x^2 + 9x - 18 = 0$$

If $x - 2$ is a factor.

$$\{\pm 3i, 2\}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 9 & -18 \\ & & 2 & 0 & 18 \\ \hline & 1 & 0 & 9 & 0 \end{array}$$

$$x^2 + 9 = 0$$

$$x = \pm 3i$$

When it is a factor, what can you say about the remainder?

Is it a factor?

$$f(x) = x^3 + x^2 + 3x + 3$$

Is $x + 2$ a factor? No

$$\begin{array}{r} -2 \overline{) 1 \quad 1 \quad 3 \quad 3} \\ \underline{-2 \quad 2 \quad -10} \\ 1 \quad -1 \quad 5 \quad -7 \end{array}$$

$$f(x) = x^3 + x^2 + 3x + 3$$

Find $f(-2)$.

$$\begin{aligned} &= (-2)^3 + (-2)^2 + 3(-2) + 3 \\ &= -7 \end{aligned}$$

$$f(x) = x^3 + x^2 + 3x + 3$$

Is $x + 3$ a factor?

No

$$f(-3) = -24$$

$$f(x) = x^3 + x^2 + 3x + 3$$

Is $x+1$ a factor?

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 3 & 3 \\ & \downarrow & -1 & 0 & -3 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

YES

Find k such that

$2x^4 + x^3 + 5x^2 - 6x + k$ $\div (x+2)$ has a remainder of 5.

$$\begin{array}{r|rrrrrr} -2 & 2 & 1 & 5 & -6 & k \\ & & -4 & 6 & -22 & 56 \\ \hline & 2 & -3 & 11 & -28 & k+56 \end{array}$$

$k+56 = 5$
 $k = -51$

$$2x^4 + x^3 + 5x^2 - 6x + k$$

Find k such that $x+2$ is a factor.

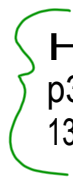
$$\begin{aligned} k+56 &= 0 \\ k &= -56 \end{aligned}$$

Remainder Theorem (summary)

The remainder of $f(x) \div (x-a)$ is $f(a)$.

Factor Theorem (summary)

The binomial $(x-a)$ is a factor of $f(x)$ iff $f(a) = 0$.



HW
p368-369
13-17, 21-27, 31-35 all odds